Sample Midterm Questions (CS 477 Spring 2013)

1. Give truth tables for each subformula of the following formulae:

   (a) $A \Rightarrow A \land (A \lor B)$

   (b) $((A \lor C) \land (B \lor C)) \Rightarrow ((A \land B) \lor C)$

   (c) $(A \Rightarrow B) \Rightarrow ((B \Rightarrow C) \Rightarrow (A \Rightarrow C))$

2. Give Natural Deduction proof trees for each of the above propositions.

3. Consider the signature $\mathcal{G} = (V = \{x, y, z\}, F = \emptyset, af = \emptyset, R = \{<\}, ar = \{< \mapsto 2\})$, and the structure $S = \{\mathcal{G}, \mathcal{D} = \mathbb{N}, \mathcal{F}, \phi, \mathcal{R}, \rho\}$ where $\rho(<)$ is normal less-than comparison. Given the formula $\forall y.\ y < x \Rightarrow (\forall z.\ \neg(z < y))$:

   (a) give an assignment that satisfies the formula, and explain why it does.

   (b) give an assignment that falsifies the formula, and explain why it does.

4. Give a Natural Deduction proof for the formula $(\exists x.\ \forall y.\ r(x, y)) \Rightarrow (\forall y.\ \exists x.\ r(x, y))$

5. Prove the following Hoare triple:

$$\{n \geq 0\}f := 1;\ i := n;\ \texttt{while}\ i > 0\ \texttt{do}\ (f := f * i;\ i := i - 1)\{f = n!\}$$

6. Give a Floyd-Hoare rule for the command $\texttt{repeat}\ C\ \texttt{until}\ B$, which repeatedly executes C until B is true. Note that B is checked *after* each execution of C, so that C is always executed at least once.

7. Calculate weakest preconditions and verification conditions for the following Hoare triples:

   (a) $\{n \geq 0\}f := 1;\ i := n;\ \texttt{while}\ i > 0\ \texttt{do}\ (f := f * i;\ i := i - 1)\{f = n!\}$

   (b) $\{a > 0 \land b > 0\}$
   $m := a;\ n := b;$
   $\texttt{while}\ n \neq m\ \texttt{do}\ (\texttt{if}\ m < n\ \texttt{then}\ n := n - m\ \texttt{else}\ m := m - n)$
   $\{a \bmod m = 0 \land b \bmod m = 0\}$